

# Supersymmetric Models and CP violation in B decays

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## Abstract

In this talk CP violation in the supersymmetric models, and especially in  $B$ -decays is discussed. We review our analysis of the supersymmetric contributions to the mixing CP asymmetries of  $B \rightarrow \phi K_S$  and  $B \rightarrow \eta' K_S$  processes. Both gluino and chargino exchanges are considered in a model independent way by using the mass insertion approximation method. The QCD factorization method is used, and parametrization of this method in terms of Wilson coefficients is presented in both decay modes. Correlations between the CP asymmetries of these processes and the direct CP asymmetry in  $b \rightarrow s\gamma$  decay are shown.

# 1 Introduction

In the Standard Model (SM), the CP violation is due to the misalignment of the mass and charged current interaction eigenstates. This misalignment is represented in the CKM matrix  $V_{CKM}$  [1], present in the charged current interaction Lagrangian,

$$L_{int}^{CC} = -\frac{g_2}{\sqrt{2}} \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma_\mu V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^+ + h.c.. \quad (1)$$

In the Wolfenstein parametrization  $V_{CKM}$  is given by

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \quad (2)$$

where the Cabibbo mixing angle  $\lambda = 0.22$ . The CKM matrix is unitary,  $V_{CKM}^\dagger V_{CKM} = 1 = V_{CKM} V_{CKM}^\dagger$ . The unitarity conditions provide strong constraints for CP violation in the Standard Model. It is, however, well known that the amount of CP violation in the Standard Model is not enough to account for the generation of the matter-antimatter asymmetry in the universe. Thus new sources for CP violation are expected from beyond the Standard Model scenarios. *E.g.* in supersymmetric models a large number of new phases emerge. These phases are strongly constrained by electric dipole moments.

The unitarity constraints can be represented as unitarity triangles, for which the length of the sides correspond to the products of elements in the CKM matrix due to the unitarity conditions. In the Standard Model, the angle  $\beta$  in the unitary triangle [2], can be measured from  $B$  meson decays. The golden mode  $B^0 \rightarrow J/\psi K_S$  is dominated by tree contribution and measurement of the CP asymmetries very accurately gives the  $\beta$  angle.

The dominant part of the decay amplitudes for  $B^0 \rightarrow \phi K_S$ ,  $\eta' K_S$  is assumed to come from the gluonic penguin, but some contribution from the tree level  $b \rightarrow u\bar{u}s$  decay is expected. The  $|\phi\rangle$  is almost pure  $|s\bar{s}\rangle$  and consequently this decay mode corresponds also accurately, up to terms of orders  $\mathcal{O}(\lambda^2)$ , to  $\sin 2\beta$  in the SM [3]. The  $b \rightarrow u\bar{u}s$  tree level contribution to  $B_d \rightarrow \eta' K$  was estimated in [4]. It was found that the tree level amplitude is less than 2% of the gluonic penguin amplitude. Thus also in this mode one measures the angle  $\beta$  with a good precision in the SM. Therefore, it is expected that NP contributions to the CP asymmetries in  $B^0 \rightarrow \phi K_S$ ,  $\eta' K_S$  decays are more significant than in  $B^0 \rightarrow J/\psi K_S$  and can compete with the SM one.

$B$  physics is a natural framework to test the beyond the Standard Model CP violation effects. It is clear that ultimately one needs to test the Standard Model with three generations and that large statistics is needed to achieve the small effects of CP violation. Flow of interesting data has been provided in recent years by the B-factories.

In this talk we will not assume any particular model for CP violation, but consider a general SUSY model. The talk is based on papers [5,6], where a comparative study

of SUSY contributions from chargino and gluino to  $B \rightarrow \phi K$  and  $B \rightarrow \eta' K$  processes in naive factorization (NF) and QCD factorization (QCDF) approaches is done. We also analyzed in [6] the branching ratios of these decays and investigated their correlations with CP asymmetries. The correlations between CP asymmetries of these processes and the direct CP asymmetry in  $b \rightarrow s\gamma$  decay [7] is also discussed in [6].

In the analysis the mass insertion method (MIA) [8] is used. Denoting by  $\Delta_{AB}^q$  the off-diagonal terms in the sfermion ( $\tilde{q} = \tilde{u}, \tilde{d}$ ) mass matrices for the up and down, respectively, where  $A, B$  indicate chirality couplings to fermions  $A, B = (L, R)$ , the A–B squark propagator can be expanded as

$$\langle \tilde{q}_A^a \tilde{q}_B^{b*} \rangle = i \left( k^2 \mathbf{1} - \tilde{m}^2 \mathbf{1} - \Delta_{AB}^q \right)_{ab}^{-1} \simeq \frac{i\delta_{ab}}{k^2 - \tilde{m}^2} + \frac{i(\Delta_{AB}^q)_{ab}}{(k^2 - \tilde{m}^2)^2} + \mathcal{O}(\Delta^2), \quad (3)$$

where  $q = u, d$  selects up or down sector, respectively,  $a, b = (1, 2, 3)$  are flavor indices,  $\mathbf{1}$  is the unit matrix, and  $\tilde{m}$  is the average squark mass. As we will see in the following, it is convenient to parametrize this expansion in terms of the dimensionless quantity  $(\delta_{AB}^q)_{ab} \equiv (\Delta_{AB}^q)_{ab}/\tilde{m}^2$ .

New physics (NP) could in principle affect the  $B$  meson decay by means of a new source of CP violating phase in the corresponding amplitude. In general this phase is different from the corresponding SM one. If so, then deviations on CP asymmetries from SM expectations can be sizeable, depending on the relative magnitude of SM and NP amplitudes. For instance, in the SM the  $B \rightarrow \phi K_S$  decay amplitude is generated at one loop and therefore it is very sensitive to NP contributions. In this respect, SUSY models with non minimal flavor structure and new CP violating phases in the squark mass matrices, can easily generate large deviations in the  $B \rightarrow \phi K_S$  asymmetry.

The time dependent CP asymmetry for  $B \rightarrow \phi K_S$  can be described by

$$a_{\phi K_S}(t) = \frac{\Gamma(\overline{B}^0(t) \rightarrow \phi K_S) - \Gamma(B(t) \rightarrow \phi K_S)}{\Gamma(\overline{B}^0(t) \rightarrow \phi K_S) + \Gamma(B(t) \rightarrow \phi K_S)} = C_{\phi K_S} \cos \Delta M_{B_d} t + S_{\phi K_S} \sin \Delta M_{B_d} t, \quad (4)$$

where  $C_{\phi K_S}$  and  $S_{\phi K_S}$  represent the direct and the mixing CP asymmetry, respectively and they are given by

$$C_{\phi K_S} = \frac{|\overline{\rho}(\phi K_S)|^2 - 1}{|\overline{\rho}(\phi K_S)|^2 + 1}, \quad S_{\phi K_S} = \frac{2\text{Im} \left[ \frac{q}{p} \overline{\rho}(\phi K_S) \right]}{|\overline{\rho}(\phi K_S)|^2 + 1}. \quad (5)$$

The parameter  $\overline{\rho}(\phi K_S)$  is defined by

$$\overline{\rho}(\phi K_S) = \frac{\overline{A}(\phi K_S)}{A(\phi K_S)}. \quad (6)$$

where  $\overline{A}(\phi K_S)$  and  $A(\phi K_S)$  are the decay amplitudes of  $\overline{B}^0$  and  $B^0$  mesons, respectively. Here, the mixing parameters  $p$  and  $q$  are defined by  $|B_1\rangle = p|B\rangle + q|\overline{B}^0\rangle$ ,  $|B_2\rangle =$

$p|B\rangle - q|\bar{B}^0\rangle$  where  $|B_{1(2)}\rangle$  are mass eigenstates of  $B$  meson. The ratio of the mixing parameters is given by

$$\frac{q}{p} = -e^{-2i\theta_d} \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}, \quad (7)$$

where  $\theta_d$  represent any SUSY contribution to the  $B - \bar{B}^0$  mixing angle. Finally, the above amplitudes can be written in terms of the matrix element of the  $\Delta B = 1$  transition as

$$\bar{A}(\phi K_S) = \langle \phi K_S | H_{eff}^{\Delta B=1} | \bar{B}^0 \rangle, \quad A(\phi K_S) = \langle \phi K_S | (H_{eff}^{\Delta B=1})^\dagger | B^0 \rangle. \quad (8)$$

Results by BaBar and Belle collaboration have been announced in [9, 10]. The experimental value of the indirect CP asymmetry parameter for  $B^0 \rightarrow J/\psi K_S$  is given by [9, 10]

$$S_{J/\psi K_S} = 0.726 \pm 0.037, \quad (9)$$

which agrees quite well with the SM prediction  $0.715_{-0.045}^{+0.055}$  [11]. Results on the corresponding  $\sin 2\beta$  extracted for  $B^0 \rightarrow \phi K_S$  process is as follows [9, 10]

$$\begin{aligned} S_{\phi K_S} &= 0.50 \pm 0.25_{-0.04}^{+0.07} \text{ (BaBar)}, \\ &= 0.06 \pm 0.33 \pm 0.09 \text{ (Belle)}, \end{aligned} \quad (10)$$

where the first errors are statistical and the second systematic. As we can see from Eq.(10), the relative central values are different. BaBar results [9] are more compatible with SM predictions, while Belle measurements [10] show a deviation from the  $c\bar{c}$  measurements of about  $2\sigma$ . Moreover, the average  $S_{\phi K_S} = 0.34 \pm 0.20$  displays  $1.7\sigma$  deviation from Eq.(9).

Furthermore, the most recent measured CP asymmetry in the  $B^0 \rightarrow \eta' K_S$  decay is found by BaBar [9] and Belle [10] collaborations as

$$\begin{aligned} S_{\eta' K_S} &= 0.27 \pm 0.14 \pm 0.03 \text{ (BaBar)} \\ &= 0.65 \pm 0.18 \pm 0.04 \text{ (Belle)}, \end{aligned} \quad (11)$$

with an average  $S_{\eta' K_S} = 0.41 \pm 0.11$ , which shows a  $2.5\sigma$  discrepancy from Eq. (9).

It is interesting to note that the results on s-penguin modes from both experiments differ from the value extracted from the  $c\bar{c}$  mode ( $J/\psi$ ), BaBar by  $2.7\sigma$  and Belle by  $2.4\sigma$  [9, 10]. At the same time the experiments agree with each other, and even the central values are quite close:

$$0.42 \pm 0.10 \text{ BaBar}, \quad 0.43_{-0.11}^{+0.12} \text{ Belle}.$$

On the other hand, the experimental measurements of the branching ratios of  $B^0 \rightarrow \phi K^0$  and  $B^0 \rightarrow \eta' K^0$  at BaBar [12], Belle [13], and CLEO [14] lead to the following averaged results [15] :

$$BR(B^0 \rightarrow \phi K^0) = (8.3_{-1.0}^{+1.2}) \times 10^{-6}, \quad (12)$$

$$BR(B^0 \rightarrow \eta' K^0) = (65.2_{-5.9}^{+6.0}) \times 10^{-6}. \quad (13)$$

From theoretical side, the SM predictions for  $BR(B \rightarrow \phi K)$  are in good agreement with Eq.(12), while showing a large discrepancy, being experimentally two to five times larger, for  $BR(B \rightarrow \eta' K)$  in Eq.(13) [16]. This discrepancy is not new and it has created a growing interest in the subject. However, since it is observed only in  $B \rightarrow \eta' K$  process, mechanisms based on the peculiar structure of  $\eta'$  meson, such as intrinsic charm and gluonium content, have been investigated to solve the puzzle. Correlations with branching ratios have been discussed in [6].

## 2 SUSY contributions to $B \rightarrow \phi(\eta')K$ decay

We first consider the supersymmetric effect in the non-leptonic  $\Delta B = 1$  processes. Such an effect could be a probe for any testable SUSY implications in CP violating experiments. The most general effective Hamiltonian  $H_{\text{eff}}^{\Delta B=1}$  for the non-leptonic  $\Delta B = 1$  processes can be expressed via the Operator Product Expansion (OPE) as [17]

$$H_{\text{eff}}^{\Delta B=1} = \left\{ \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) \right\} \\ + \left\{ Q_i \rightarrow \tilde{Q}_i, C_i \rightarrow \tilde{C}_i \right\}, \quad (14)$$

where  $\lambda_p = V_{pb} V_{ps}^*$ , with  $V_{pb}$  the unitary CKM matrix elements satisfying the unitarity triangle relation  $\lambda_t + \lambda_u + \lambda_c = 0$ , and  $C_i \equiv C_i(\mu_b)$  are the Wilson coefficients at low energy scale  $\mu_b \simeq \mathcal{O}(m_b)$ . The basis  $Q_i \equiv Q_i(\mu_b)$  is given by the relevant local operators renormalized at the same scale  $\mu_b$ , namely

$$\begin{aligned} Q_2^p &= (\bar{p}b)_{V-A} (\bar{s}p)_{V-A}, & Q_1^p &= (\bar{p}_\alpha b_\beta)_{V-A} (\bar{s}_\beta p_\alpha)_{V-A} \\ Q_3 &= (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V-A}, & Q_4 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \\ Q_5 &= (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V+A}, & Q_6 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}, \\ Q_7 &= (\bar{s}b)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}q)_{V+A}, & Q_8 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}_\beta q_\alpha)_{V+A}, \\ Q_9 &= (\bar{s}b)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}q)_{V-A}, & Q_{10} &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q \frac{3}{2} e_q (\bar{q}_\beta q_\alpha)_{V-A}, \\ Q_{7\gamma} &= \frac{e}{8\pi^2} m_b \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) F_{\mu\nu} b, & Q_{8g} &= \frac{g_s}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) G_{\mu\nu}^A t_{\alpha\beta}^A b_\beta. \end{aligned} \quad (15)$$

Here  $\alpha$  and  $\beta$  stand for color indices, and  $t_{\alpha\beta}^A$  for the  $SU(3)_c$  color matrices,  $\sigma^{\mu\nu} = \frac{1}{2}i[\gamma^\mu, \gamma^\nu]$ . Moreover,  $e_q$  are quark electric charges in unity of  $e$ ,  $(\bar{q}q)_{V\pm A} \equiv \bar{q}\gamma_\mu(1 \pm \gamma_5)q$ , and  $q$  runs over  $u, d, s$ , and  $b$  quark labels. In the SM only the first part of right hand side of Eq.(14) (inside first curly brackets) containing operators  $Q_i$  will contribute, where  $Q_{1,2}^p$  refer to the current-current operators,  $Q_{3-6}$  to the QCD penguin operators, and  $Q_{7-10}$  to

the electroweak penguin operators, while  $Q_{7\gamma}$  and  $Q_{8g}$  are the magnetic and the chromomagnetic dipole operators, respectively. In addition, operators  $\tilde{Q}_i \equiv \tilde{Q}_i(\mu_b)$  are obtained from  $Q_i$  by the chirality exchange  $(\bar{q}_1 q_2)_{V\pm A} \rightarrow (\bar{q}_1 q_2)_{V\mp A}$ . Notice that in the SM the coefficients  $\tilde{C}_i$  identically vanish due to the V-A structure of charged weak currents, while in MSSM they can receive contributions from both chargino and gluino exchanges [18, 19].

As mentioned, we calculated in [5] the chargino contribution to the Wilson coefficients in the MIA approximation. In MIA framework, one chooses a basis (called super-CKM basis) where the couplings of fermions and sfermions to neutral gaugino fields are flavor diagonal. In this basis, the interacting Lagrangian involving charginos is given by

$$\begin{aligned} \mathcal{L}_{q\tilde{q}\tilde{\chi}^+} = & -g \sum_k \sum_{a,b} \left( V_{k1} K_{ba}^* \bar{d}_L^a (\tilde{\chi}_k^+)^* \tilde{u}_L^b - U_{k2}^* (Y_d^{\text{diag}} \cdot K^+)_{ab} \bar{d}_R^a (\tilde{\chi}_k^+)^* \tilde{u}_L^b \right. \\ & \left. - V_{k2} (K \cdot Y_u^{\text{diag}})_{ab} \bar{d}_L^a (\tilde{\chi}_k^+)^* \tilde{u}_R^b \right), \end{aligned} \quad (16)$$

where  $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$ , and contraction of color and Dirac indices is understood. Here  $Y_{u,d}^{\text{diag}}$  are the diagonal Yukawa matrices, and  $K$  stands for the CKM matrix. The indices  $a, b$  and  $k$  label flavor and chargino mass eigenstates, respectively, and  $V, U$  are the chargino mixing matrices defined by

$$U^* M_{\tilde{\chi}^+} V^{-1} = \text{diag}(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}), \text{ and } M_{\tilde{\chi}^+} = \begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & \mu \end{pmatrix}, \quad (17)$$

where  $M_2$  is the weak gaugino mass,  $\mu$  is the supersymmetric Higgs mixing term, and  $\tan \beta$  is the ratio of the vacuum expectation value (VEV) of the up-type Higgs to the VEV of the down-type Higgs<sup>1</sup>. As one can see from Eq.(16), the higgsino couplings are suppressed by Yukawas of the light quarks, and therefore they are negligible, except for the stop-bottom interaction which is directly enhanced by the top Yukawa ( $Y_t$ ). In our analysis we neglect the higgsino contributions proportional to the Yukawa couplings of light quarks with the exception of the bottom Yukawa  $Y_b$ , since its effect could be enhanced by large  $\tan \beta$ . However, it is easy to show that this vertex cannot affect dimension six operators of the effective Hamiltonian for  $\Delta B = 1$  transitions (operators  $Q_{i=1-10}$  in Eq.(14)) and only interactions involving left down quarks will contribute. On the contrary, contributions proportional to bottom Yukawa  $Y_b$  enter in the Wilson coefficients of dipole operators ( $C_{7\gamma}, C_{8g}$ ) due to the chirality flip of  $b \rightarrow s\gamma$  and  $b \rightarrow sg$  transitions.

The calculation of  $B \rightarrow \phi(\eta')K$  decays involves the evaluation of the hadronic matrix elements of related operators in the effective Hamiltonian, which is the most uncertain part of this calculation. In the limit in which  $m_b \gg \Lambda_{QCD}$  and neglecting QCD corrections in  $\alpha_s$ , the hadronic matrix elements of B meson decays in two mesons can be factorized, for example for  $B \rightarrow M_1 M_2$ , in the form

$$\langle M_1 M_2 | Q_i | \bar{B}^0 \rangle = \langle M_1 | j_1 | \bar{B}^0 \rangle \times \langle M_2 | j_2 | 0 \rangle \quad (18)$$

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<sup>1</sup>This  $\tan \beta$  should not be confused with the angle  $\beta$  of the unitarity triangle.

where  $M_{1,2}$  indicates two generic mesons,  $Q_i$  are local four fermion operators of the effective Hamiltonian in Eq.(14), and  $j_{1,2}$  represent bilinear quark currents. Then, the final results can be usually parametrized by the product of the decay constants and the transition form factors. This approach is known as naive factorization (NF) [20, 21]. In QCDF the hadronic matrix element for  $B \rightarrow MK$  with  $M = \phi, \eta'$  in the heavy quark limit  $m_b \gg \Lambda_{QCD}$  can be written as [22]

$$\langle MK|Q_i|B\rangle_{QCDF} = \langle MK|Q_i|B\rangle_{NF} \cdot \left[ 1 + \sum_n r_n \alpha_S^n + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right) \right], \quad (19)$$

where  $\langle MK|Q_i|B\rangle_{NF}$  denotes the NF results. The second and third term in the bracket represent the radiative corrections in  $\alpha_S$  and  $\Lambda_{QCD}/m_b$  expansions, respectively. Notice that, even though at higher order in  $\alpha_s$  the simple factorization is broken, these corrections can be calculated systematically in terms of short-distance coefficients and meson light-cone distribution functions.

In the QCD factorization method [22, 23], the decay amplitudes of  $B \rightarrow \phi(\eta')K$  can be expressed as

$$\mathcal{A}(B \rightarrow \phi(\eta')K) = \mathcal{A}^f(B \rightarrow \phi(\eta')K) + \mathcal{A}^a(B \rightarrow \phi(\eta')K), \quad (20)$$

where

$$\mathcal{A}^f(B \rightarrow \phi(\eta')K) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{s=1}^{10} V_{pb} V_{ps}^* a_i^{\phi(\eta')} \langle \phi(\eta')K|Q_i|B\rangle_{NF}, \quad (21)$$

and

$$\mathcal{A}^a(B \rightarrow \phi(\eta')K) = \frac{G_F}{\sqrt{2}} f_B f_K f_\phi \sum_{p=u,c} \sum_{i=1}^{10} V_{pb} V_{ps}^* b_i^{\phi(\eta')}. \quad (22)$$

The first term  $\mathcal{A}^f(B \rightarrow \phi(\eta')K)$  includes vertex corrections, penguin corrections and hard spectator scattering contributions which are involved in the parameters  $a_i^{\phi(\eta')}$ . The other term  $\mathcal{A}^a(B \rightarrow \phi(\eta')K)$  includes the weak annihilation contributions which are absorbed in the parameters  $b_i^{\phi(\eta')}$ . However, these contributions contain infrared divergences, and the subtractions of these divergences are usually parametrized as [23]

$$\int_0^1 \frac{dx}{x} \rightarrow X_{H,A} \equiv \left( 1 + \rho_{H,A} e^{i\phi_{H,A}} \right) \ln \left( \frac{m_B}{\Lambda_{QCD}} \right), \quad (23)$$

where  $\rho_{H,A}$  are free parameters expected to be of order of  $\rho_{H,A} \simeq \mathcal{O}(1)$ , and  $\phi_{H,A} \in [0, 2\pi]$ . As already discussed in Ref.[23], if one does not require fine tuning of the annihilation phase  $\phi_A$ , the  $\rho_A$  parameter gets an upper bound from measurements on branching ratios, which is of order of  $\rho_A \lesssim 2$ . Clearly, large values of  $\rho_A$  are still possible, but in this case strong fine tuning in the phase  $\phi_A$  is required. However, assumptions of very large values of  $\rho_{H,A}$ , which implicitly means large contributions from hard scattering and weak annihilation diagrams, seem to be quite unrealistic. In [6] we assumed  $\rho < 2$ .

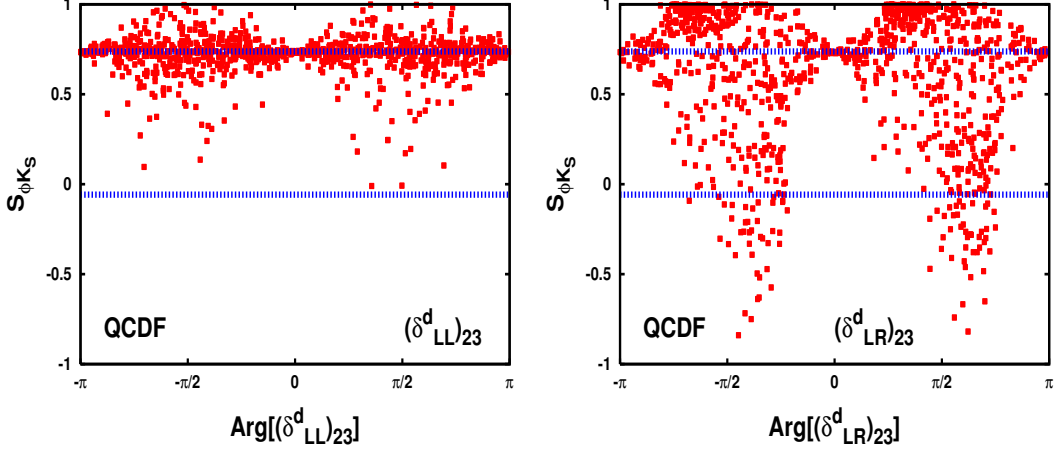


Figure 1:  $S_{\phi K_S}$  as a function of  $\arg[(\delta_{LL}^d)_{23}]$  (left) and  $\arg[(\delta_{LR}^d)_{23}]$  (right) with gluino contribution of one mass insertion. The region inside the two horizontal lines corresponds to the allowed experimental region at  $2\sigma$  level.

### 3 CP asymmetry in $B \rightarrow \phi K_S$ and in $B \rightarrow \eta' K_S$

In order to simplify our analysis, it is useful to parametrize the SUSY effects by introducing the ratio of SM and SUSY amplitudes as follows

$$\left( \frac{A^{\text{SUSY}}}{A^{\text{SM}}} \right)_{\phi K_S} \equiv R_\phi e^{i\theta_\phi} e^{i\delta_\phi} \quad (24)$$

$$(25)$$

and analogously for the  $\eta' K_S$  decay mode

$$\left( \frac{A^{\text{SUSY}}}{A^{\text{SM}}} \right)_{\eta' K_S} \equiv R_{\eta'} e^{i\theta_{\eta'}} e^{i\delta_{\eta'}} \quad (26)$$

where  $R_i$  stands for the corresponding absolute values of  $|\frac{A^{\text{SUSY}}}{A^{\text{SM}}}|$ , the angles  $\theta_\phi, \eta'$  are the corresponding SUSY CP violating phase, and  $\delta_\phi, \eta' = \delta_\phi^{\text{SM}} - \delta_\phi^{\text{SUSY}}$  parametrize the strong (CP conserving) phases. In this case, the mixing CP asymmetry  $S_{\phi K_S}$  in Eq.(4) takes the following form

$$S_{\phi K_S} = \frac{\sin 2\beta + 2R_\phi \cos \delta_\phi \sin(\theta_\phi + 2\beta) + R_\phi^2 \sin(2\theta_\phi + 2\beta)}{1 + 2R_\phi \cos \delta_\phi \cos \theta_\phi + R_\phi^2}. \quad (27)$$

and analogously for  $B \rightarrow \eta' K_S$

$$S_{\eta' K_S} = \frac{\sin 2\beta + 2R_{\eta'} \cos \delta_{\eta'} \sin(\theta_{\eta'} + 2\beta) + R_{\eta'}^2 \sin(2\theta_{\eta'} + 2\beta)}{1 + 2R_{\eta'} \cos \delta_{\eta'} \cos \theta_{\eta'} + R_{\eta'}^2}. \quad (28)$$



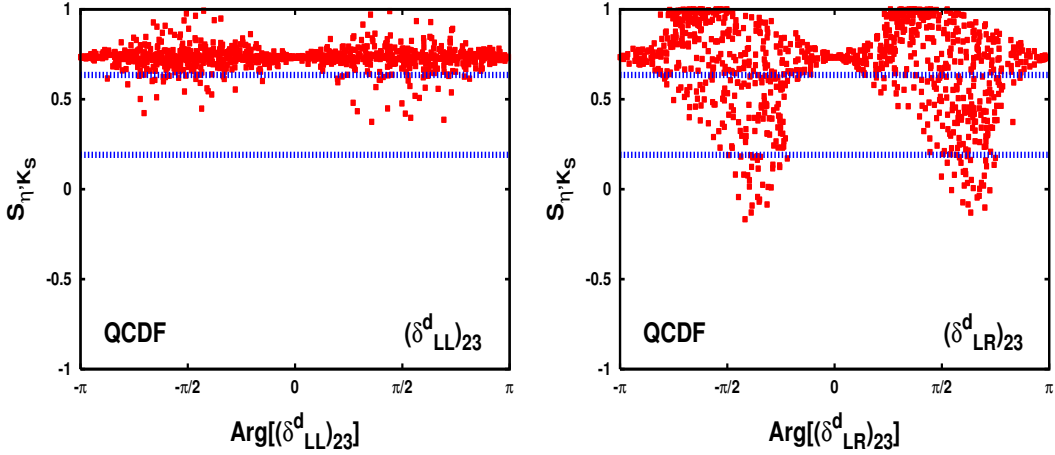


Figure 2: As in Fig. 1, but for  $S_{\eta' K_S}$ .

Our numerical results for the gluino contributions to CP asymmetry  $S_{\Phi K_S}$  are presented in Fig. 1, and to CP asymmetry  $S_{\eta' K_S}$  are presented in Fig. 2. In all the plots, regions inside the horizontal lines indicate the allowed  $2\sigma$  experimental range. In the plots only one mass insertion per time is taken active, in particular this means that we scanned over  $|(\delta_{LL}^d)_{23}| < 1$  and  $|(\delta_{LR}^d)_{23}| < 1$ . Then,  $S_{\Phi(\eta') K_S}$  is plotted versus  $\theta_\phi$ , which in the case of one dominant mass insertion should be identified here as  $\theta_\phi = \arg[(\delta_{AB}^d)_{ij}]$ .

We have scanned over the relevant SUSY parameter space, in this case the average squark mass  $\tilde{m}$  and gluino mass  $m_{\tilde{g}}$ , assuming SM central values [24]. Moreover, we require that the SUSY spectra satisfy the present experimental lower mass bounds [24]. In particular,  $m_{\tilde{g}} > 200$  GeV,  $\tilde{m} > 300$  GeV. In addition, we impose that the branching ratio (BR) of  $b \rightarrow s\gamma$  and the  $B - \bar{B}$  mixing are satisfied at 95% C.L. [25], namely  $2 \times 10^{-4} \leq BR(b \rightarrow s\gamma) < 4.5 \times 10^{-4}$ . Then, the allowed ranges for  $|(\delta_{LL}^d)_{23}|$  and  $|(\delta_{LR}^d)_{23}|$  are obtained by taken into account the above constraints on  $b \rightarrow s\gamma$  and  $B - \bar{B}$  mixing. We have also scanned over the full range of the parameters  $\rho_{A,H}$  and  $\phi_{A,H}$  in  $X_A$  and  $X_H$ , respectively, as defined in Eq.(23).

The chargino effects to  $S_{\Phi K_S}$  and  $S_{\eta' K_S}$  [5, 6] are summarized in Fig. 3 and Fig. 4, where  $S_{\phi(\eta') K_S}$  is plotted versus the argument of the relevant chargino mass insertions, namely  $(\delta_{LL}^u)_{32}$  and  $(\delta_{RL}^u)_{32}$ .

As in the gluino dominated scenario, we have scanned over the relevant SUSY parameter space, in particular, the average squark mass  $\tilde{m}$ , weak gaugino mass  $M_2$ , the  $\mu$  term, and the light right stop mass  $\tilde{m}_{\tilde{t}_R}$ . Also  $\tan \beta = 40$  has been assumed and we take into account the present experimental bounds on SUSY spectra, in particular  $\tilde{m} > 300$  GeV, the lightest chargino mass  $M_\chi > 90$  GeV, and  $\tilde{m}_{\tilde{t}_R} \geq 150$  GeV. As in the gluino case, we scan over the real and imaginary part of the mass insertions  $(\delta_{LL}^u)_{32}$  and  $(\delta_{RL}^u)_{32}$ , by considering the constraints on  $BR(b \rightarrow s\gamma)$  and  $B - \bar{B}$  mixing at 95% C.L.. The  $b \rightarrow s\gamma$  constraints

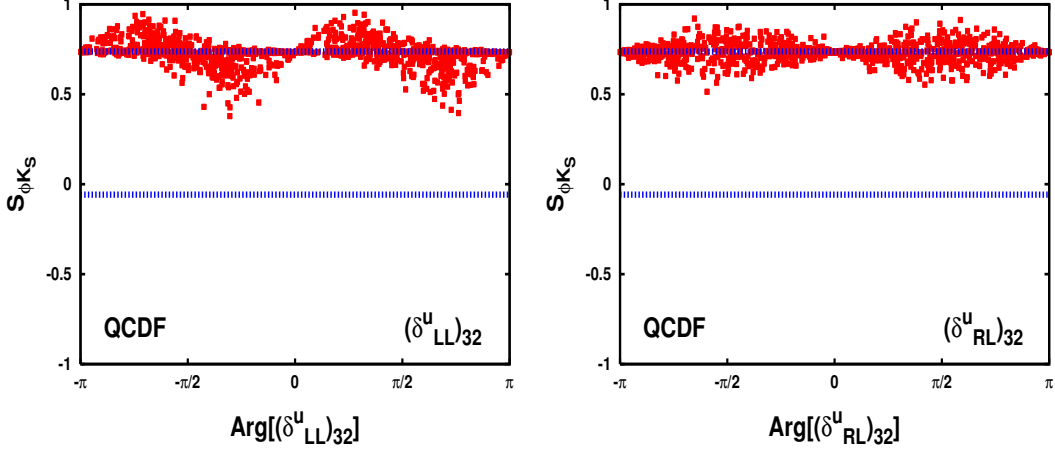


Figure 3: As in Fig. 1, but for  $S_{\phi K_S}$  as a function of  $\arg[(\delta^u_{LL})_{32}]$  (left) and  $\arg[(\delta^u_{RL})_{32}]$  (right) with chargino contribution of one mass insertion .

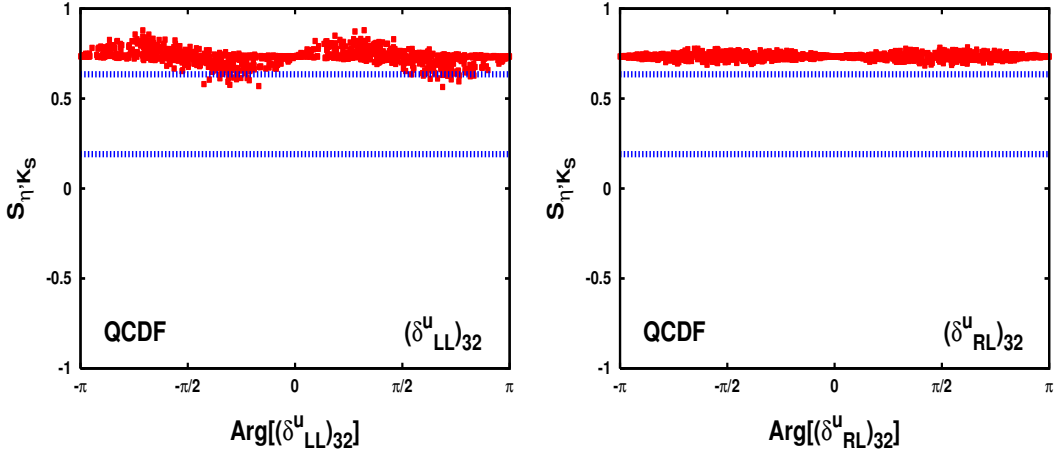


Figure 4: As in Fig. 3, but for  $S_{\eta' K_S}$ .

impose stringent bounds on  $(\delta^u_{LL})_{32}$ , specially at large  $\tan\beta$  [5]. Finally, as in the other plots, we scanned over the QCDF free parameters  $\rho_{A,H} < 2$  and  $0 < \phi_{A,H} < 2\pi$ .

The reason why extensive regions of negative values of  $S_{\phi K_S}$  are excluded here, is only due to the  $b \rightarrow s\gamma$  constraints [5]. Indeed, the inclusion of  $(\delta^u_{LL})_{32}$  mass insertion can generate large and negative values of  $S_{\phi K_S}$ , by means of chargino contributions to chromo-magnetic operator  $Q_{8g}$  which are enhanced by terms of order  $m_{\chi^\pm}/m_b$ . However, contrary to the gluino scenario, the ratio  $|C_{8g}/C_{7\gamma}|$  is not enhanced by color factors and large contributions to  $C_{8g}$  leave unavoidably to the breaking of  $b \rightarrow s\gamma$  constraints.

We plot in Figs. 5 the correlations between  $S_{\phi K_S}$  versus  $S_{\eta' K_S}$  for both chargino and gluino in QCDF. For illustrative purposes, in all figures analyzing correlations, we colored

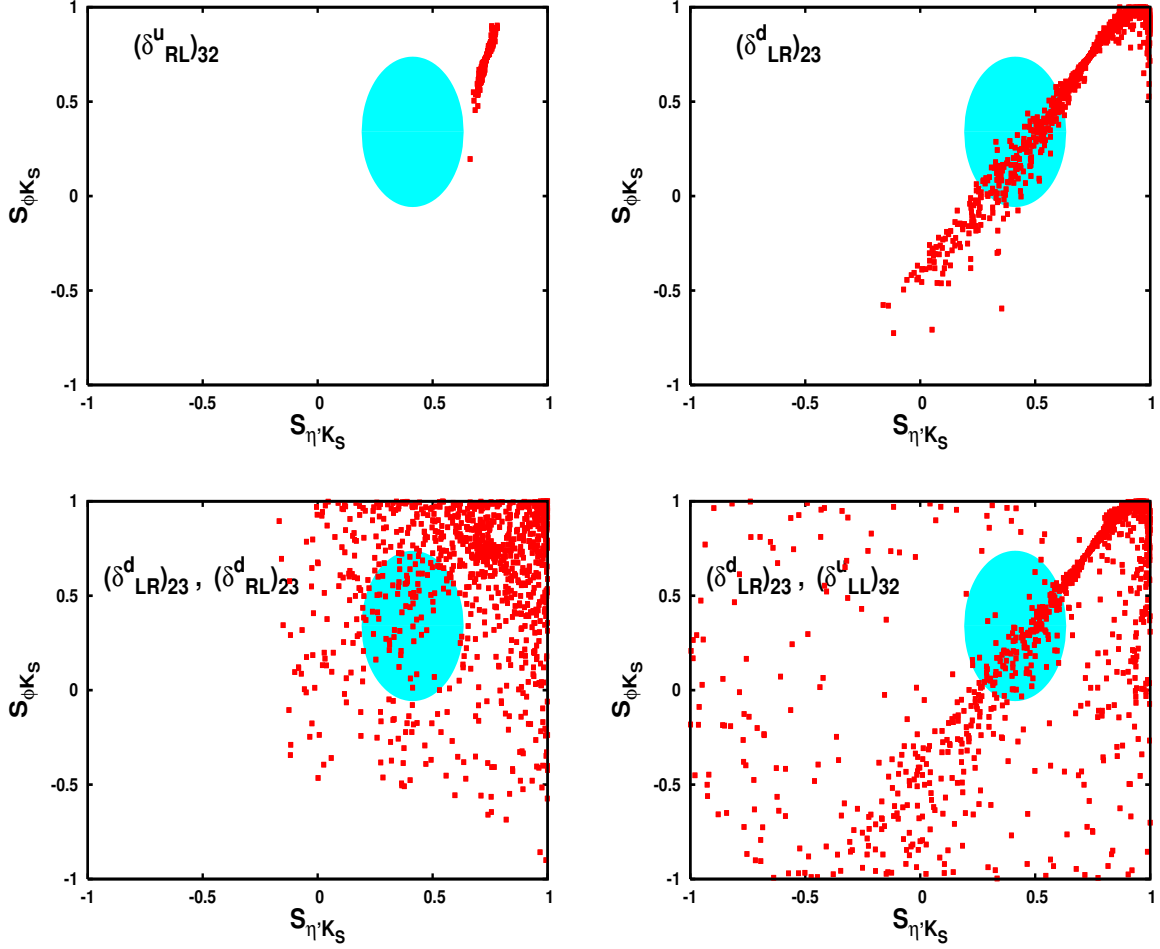


Figure 5: Above: Correlation of asymmetries  $S_{\Phi K_S}$  versus  $S_{\eta' K_S}$  with the contribution of one mass insertion  $(\delta_{RL}^u)_{32}$  (left) and  $(\delta_{LR}^d)_{23}$  (right), for chargino (left) and gluino (right) exchanges. Region inside the ellipse corresponds to the allowed experimental ranges at  $2\sigma$  level. Below: as previously, but for  $\arg[(\delta_{LR}^d)_{23}] = \arg[(\delta_{RL}^d)_{23}]$  (left) and  $\arg[(\delta_{LR}^d)_{23}] = \arg[(\delta_{LL}^u)_{32}]$  (right), with the contribution of two mass insertions  $(\delta_{LR}^d)_{23}$  &  $(\delta_{RL}^d)_{23}$  (left) for gluino exchanges, and  $(\delta_{LR}^d)_{23}$  &  $(\delta_{LL}^u)_{32}$  (right) for both gluino and chargino exchanges.

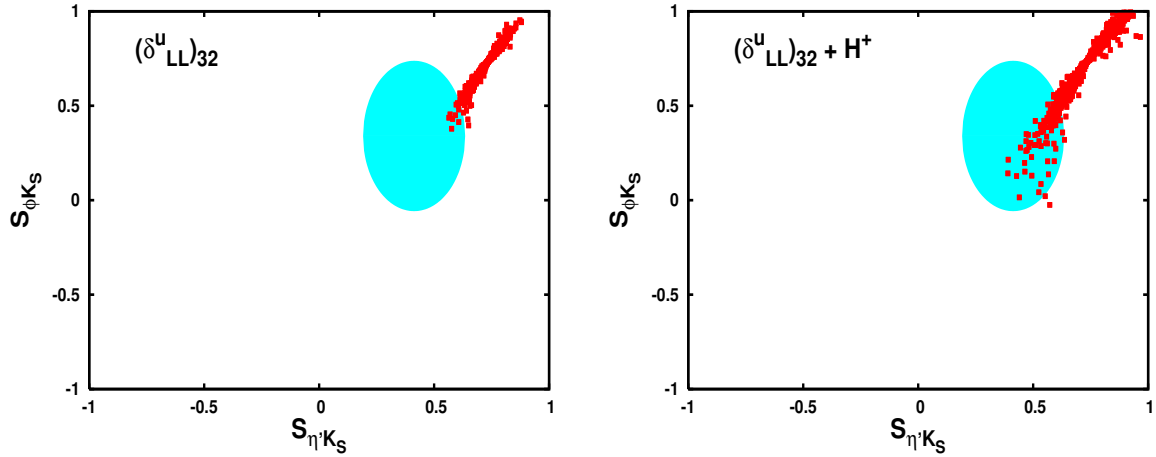


Figure 6: Correlation of asymmetries  $S_{\phi K_S}$  versus  $S_{\eta' K_S}$  for chargino contribution with single mass insertion  $(\delta_{LL}^u)_{32}$ . In the right plot the effect of a charged Higgs exchange, with mass  $m_H = 200$  GeV and  $\tan \beta = 40$ , has been taken into account.

the area of the ellipse corresponding to the allowed experimental range at  $2\sigma$  level.<sup>2</sup>

In Fig. 6 the impact of a light charged Higgs in chargino exchanges is presented, when a charged Higgs with mass  $m_H = 200$  GeV and  $\tan \beta = 40$  has been taken into account. The effects of charged Higgs exchange in the case of  $(\delta_{RL}^u)_{32}$  mass insertion are negligible, as we expect from the fact that terms proportional to  $(\delta_{RL}^u)_{32}$  in  $b \rightarrow s\gamma$  and  $b \rightarrow sg$  amplitudes are not enhanced by  $\tan \beta$ . On the other hand, in gluino exchanges with  $(\delta_{LR}^d)_{23}$  or  $(\delta_{LL}^d)_{23}$ , the most conspicuous effect of charged Higgs contribution is in populating the area outside the allowed experimental region. This is due to a destructive interference with  $b \rightarrow s\gamma$  amplitude, which relaxes the  $b \rightarrow s\gamma$  constraints. The most relevant effect of a charged Higgs exchange is in the scenario of chargino exchanges with  $(\delta_{LL}^u)_{32}$  mass insertion. In this case, as can be seen from Fig. 6, a strong destructive interference with  $b \rightarrow s\gamma$  amplitude can relax the  $b \rightarrow s\gamma$  constraints in the right direction, allowing chargino predictions to fit inside the experimental region. Moreover, we have checked that, for  $\tan \beta = 40$ , charged Higgs heavier than approximately 600 GeV cannot affect the CP asymmetries significantly.

## 4 Direct CP asymmetry in $b \rightarrow s\gamma$ versus $S_{\phi(\eta')K_S}$

Next we present the correlation for SUSY predictions between the direct CP asymmetry  $A_{CP}(b \rightarrow s\gamma)$  in  $b \rightarrow s\gamma$  decay and the other ones in  $B \rightarrow \phi(\eta')K_S$ . The CP asymmetry

<sup>2</sup>All ellipses here have axes of length  $4\sigma$ . As a first approximation, no correlation between the expectation values of the two observables have been assumed.

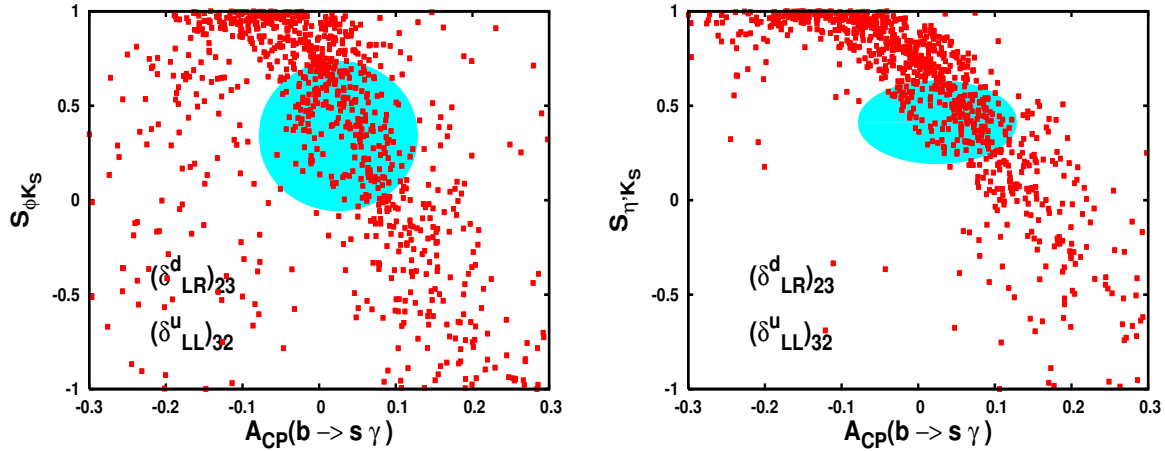


Figure 7: Correlations of  $S_{\phi K_S}$  versus  $A_{CP}(b \rightarrow s\gamma)$  (left) and  $S_{\eta' K_S}$  versus  $A_{CP}(b \rightarrow s\gamma)$  (right), for gluino and chargino contributions with mass insertions  $(\delta_{LR}^d)_{23}$  and  $(\delta_{LL}^u)_{32}$  respectively.

in  $b \rightarrow s\gamma$  is measured in the inclusive radiative decay of  $B \rightarrow X_s\gamma$  by the quantity

$$A_{CP}(b \rightarrow s\gamma) = \frac{\Gamma(\bar{B} \rightarrow X_s\gamma) - \Gamma(B \rightarrow X_{\bar{s}}\gamma)}{\Gamma(\bar{B} \rightarrow X_s\gamma) + \Gamma(B \rightarrow X_{\bar{s}}\gamma)}. \quad (29)$$

The SM prediction for  $A_{CP}(b \rightarrow s\gamma)$  is very small, less than 1% in magnitude, but known with high precision [7]. Indeed, inclusive decay rates of B mesons are free from large theoretical uncertainties since they can be reliably calculated in QCD using the OPE. Thus, the observation of sizeable effects in  $A_{CP}(b \rightarrow s\gamma)$  would be a clean signal of new physics. In particular, large asymmetries are expected in models with enhanced chromomagnetic dipole operators, like for instance supersymmetric models [7].

In Fig. 7 we show our results for two mass insertions  $(\delta_{LR}^d)_{23}$  and  $(\delta_{LL}^u)_{32}$  with both gluino and chargino exchanges. In this case we see that  $S_{\phi K_S}$  constraints do not set any restriction on  $A_{CP}(b \rightarrow s\gamma)$ , and also large and positive values of  $A_{CP}(b \rightarrow s\gamma)$  asymmetry can be achieved. However, by imposing the constraints on  $S_{\eta' K_S}$ , see plot on the right side of Fig. 7, the region of negative  $A_{CP}(b \rightarrow s\gamma)$  is disfavored in this scenario as well.

## 5 Conclusions

CP violation remains as one of the most interesting research topics both theoretically and experimentally. Especially the  $B$ -meson decay modes seem to be ideally suited for searching effects of new physics, since new interesting results are mounting from the  $B$ -factories. For interpretation of those results, which get more accurate with more statistics, also reducing theoretical uncertainties in the calculations is a big challenge.

The  $B$ -meson decays to  $\phi K$ ,  $\eta' K$ , and to  $X_S \gamma$  provide a clean window to the physics beyond the SM. Here our analysis of the supersymmetric contributions to the CP asymmetries and the branching ratios of these processes in a model independent way has been reviewed. The relevant SUSY contributions in the  $b \rightarrow s$  transitions, namely chargino and gluino exchanges in box and penguin diagrams, have been considered by using the mass insertion method.

Due to the stringent constraints from the experimental measurements of  $BR(b \rightarrow s \gamma)$ , the scenario with pure chargino exchanges cannot give large and negative values for CP asymmetry  $S_{\phi K_S}$ . It is, however, seen that charged Higgs may enhance the chargino contributions substantially. On the other hand, it is quite possible for gluino exchanges to account for  $S_{\phi K_S}$  and  $S_{\eta' K_S}$  at the same time.

We also discussed the correlations between the CP asymmetries of these processes and the direct CP asymmetry in  $b \rightarrow s \gamma$  decay. In this case, we found that the general trend of SUSY models, satisfying all the experimental constraints, favors large and positive contributions to  $b \rightarrow s \gamma$  asymmetry.

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